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$B \to \phi K^*$, $B_d^0 \to \phi K_S$ and New Physics

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Abstract

We consider the new-physics (NP) solution to the polarization puzzle in $B \to \phi K^*$ decays. We note that any such solution must reproduce the data in $B_d^0 \to \phi K_S$, where there are disagreements with the standard model in CP-asymmetry measurements. We examine 10 NP operators, of S/P, V/A and T variety. We find that, as long as $B_d^0 \to \phi K_S$ exhibits large CP-violating effects, no single operator can explain the observations in both $B \to \phi K^*$ and $B_d^0 \to \phi K_S$. For 2-NP-operator solutions, there are four possibilities, all of S/P type, which are presently allowed. We discuss ways of distinguishing among these solutions in the future. Models which contain only V/A operators, such as those with supersymmetry or extra Z' bosons, cannot explain both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ data. On the other hand, the two-Higgs-doublet model, which has only S/P operators, is favored.

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$B \to V_1 V_2$ decays (the V_i are vector mesons) really represent three transitions when the spins of the V_i are taken into account. That is, the final state can be transversely (2 states) or longitudinally (1 state) polarized. A naive calculation within the standard model (SM) shows that the transverse amplitudes are suppressed by a factor of size m_V/m_B (V is one of the vector mesons) with respect to the longitudinal amplitude. One then expects the fraction of transverse decays, f_T, to be much less than the fraction of longitudinal decays, f_L.

However, it was observed that these two fractions are roughly equal in the decay $B \to \phi K^*$: $f_T/f_L \simeq 1$ [1, 2, 3]. This is the "polarization puzzle." If one goes beyond the naive SM, there are two explanations [4] which account for this surprising result: penguin annihilation (PA) [5] and non-perturbative rescattering [6, 7]. Still, there are question marks associated with both of these. First, PA is a subleading amplitude that is power suppressed by $O(1/m_b)$. Second, for rescattering, it is not obvious whether such a non-perturbative effect is of leading or subleading order. Hence, the SM explanations of the large f_T/f_L generally require enhanced subleading amplitudes (certain for PA; possible for rescattering).

One can also explain the f_T/f_L measurement by introducing physics beyond the SM. Suppose there are new-physics (NP) contributions to the $\bar{b} \to \bar{s}s\bar{s}$ quark-level amplitude. If their form is chosen correctly, these will contribute dominantly to f_T in $B \to \phi K^*$ and not to f_L, so that one can reproduce the measured value of f_T/f_L if the NP amplitude has the right size [8]. In this paper, we explore the NP explanation – we assume that neither PA nor non-perturbative rescattering produce dominant contributions to the transverse amplitudes, and are therefore not the explanation of the measurement of a large f_T/f_L.

BR	$8.3^{+1.2}_{-1.0} \times 10^{-6}$
S_{CP}	$0.44^{+0.17}_{-0.18}$
A_{CP}	0.23 ± 0.15

Table 1: Measurements of $B_d^0 \to \phi K_S$. Included are the branching fraction (BR) [9, 10, 11], the indirect (mixing-induced) CP asymmetry (S_{CP}) [11, 12], and the direct CP symmetry (A_{CP}) [11, 12].

Now, any NP contribution to $\bar{b} \to \bar{s}s\bar{s}$ will also affect $B_d^0 \to \phi K_S$. Thus, any constraints on such NP must take into account the measurements of both $B \to \phi K^*$ and $B_d^0 \to \phi K_S$. The $B_d^0 \to \phi K_S$ data are shown in Table 1. The CP-violating observables are particularly intriguing. Within the SM, including small corrections, the indirect (mixing-induced) CP asymmetry in $B_d^0 \to \phi K_S$ (S_{CP}) is expected to be a bit larger than that in charmonium B_d^0 decays [13], found to be $S_{CP}(\text{charmonium}) = 0.672 \pm 0.024$ [11]. In addition, the direct CP asymmetry in $B_d^0 \to \phi K_S$ (A_{CP}) is expected to vanish. In other words, the central values of both of these measurements exhibit disagreements with the expectations of the SM. This provides a hint of NP in $\bar{b} \to \bar{s}$ transitions[4].

[4] A more significant hint (signal?) of NP, also in $\bar{b} \to \bar{s}$ transitions, is provided by $B \to \pi K$ decays,

On the other hand, the errors are sufficiently large that the discrepancies are only at the level of $\lesssim 2\sigma$. This means that the hint of NP is not statistically significant. It also means that any constraints on NP in $B \to \phi K^*$ are not that strong. For this reason, in this paper we also perform the analysis in the scenario in which the hint of NP in $B_d^0 \to \phi K_S$ becomes a true signal in the future. That is, in this case we assume that the future measurements of S_{CP} and A_{CP} stay at their present central values, but the errors are reduced by a factor of 2. If the discrepancies with the SM in $B_d^0 \to \phi K_S$ get more pronounced, it will be necessary to consider the results from this second scenario.

We assume that there is a NP contribution to $\bar{b} \to \bar{s}s\bar{s}$. We consider operators of the form

$$
\begin{aligned}
O_{LL}^{V/A} &= \bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{s}\gamma^\mu(1-\gamma_5)s \ , \\
O_{LR}^{V/A} &= \bar{s}\gamma_\mu(1-\gamma_5)b\,\bar{s}\gamma^\mu(1+\gamma_5)s \ , \\
O_{RL}^{V/A} &= \bar{s}\gamma_\mu(1+\gamma_5)b\,\bar{s}\gamma^\mu(1-\gamma_5)s \ , \\
O_{RR}^{V/A} &= \bar{s}\gamma_\mu(1+\gamma_5)b\,\bar{s}\gamma^\mu(1+\gamma_5)s \ , \\
O_{LL}^{S/P} &= \bar{s}(1-\gamma_5)b\,\bar{s}(1-\gamma_5)s \ , \\
O_{LR}^{S/P} &= \bar{s}(1-\gamma_5)b\,\bar{s}(1+\gamma_5)s \ , \\
O_{RL}^{S/P} &= \bar{s}(1+\gamma_5)b\,\bar{s}(1-\gamma_5)s \ , \\
O_{RR}^{S/P} &= \bar{s}(1+\gamma_5)b\,\bar{s}(1+\gamma_5)s \ , \\
O_{L}^{T} &= \bar{s}\sigma_{\mu\nu}(1-\gamma_5)b\,\bar{s}\sigma^{\mu\nu}(1-\gamma_5)s \ , \\
O_{R}^{T} &= \bar{s}\sigma_{\mu\nu}(1+\gamma_5)b\,\bar{s}\sigma^{\mu\nu}(1+\gamma_5)s \ . \qquad (1)
\end{aligned}
$$

In the above operators, we take the colors of the quark fields in each current to be the same. This is the case in most typical NP models (multi-Higgs-doublets, supersymmetry, extra Z's, etc.). For S/P operators, a Fierz transformation of the fermions and colors is required in order to get a non-vanishing contribution to the production of the final-state vector meson ϕ from the vacuum (within factorization):

$$
\begin{aligned}
O_{RR(LL)}^{S/P} &= -\frac{1}{2N_c}\bar{s}(1\pm\gamma_5)b\,\bar{s}(1\pm\gamma_5)s - \frac{1}{8N_c}\bar{s}\sigma_{\mu\nu}(1\pm\gamma_5)b\,\bar{s}\sigma^{\mu\nu}(1\pm\gamma_5)s \ , \\
O_{RL(LR)}^{S/P} &= -\frac{1}{2N_c}\bar{s}\gamma_\mu(1\pm\gamma_5)b\,\bar{s}\gamma^\mu(1\mp\gamma_5)s \ . \qquad (2)
\end{aligned}
$$

(We have neglected the octet piece coming from the color Fierz transformation, which is justified within factorization.)

There are also 10 operators in which one has different quark colors in the currents. However, these can be obtained from Eq. (1) as follows. Suppose that there is only one type of Lorentz structure. The effective Hamiltonian can then be written

$$
H_{eff} = B_1 \bar{s}O_1 b \bar{s}O_2 s + B_2 \bar{s}_\alpha O_1 b_\beta \bar{s}_\beta O_2 s_\alpha \ , \qquad (3)
$$

where the disagreement with the SM has reached the 5σ level [11], assuming that $|C/T|$ is small, as is expected in the SM [14] (C and T are diagrams contributing to the decay [15]).

where $B_{1,2}$ are complex coefficients, $O_{1,2}$ represent the Lorentz structure (S/P, V/A or T), and α, β are color indices. We will call the operator associated with B_1 "color allowed" and that associated with B_2 "color suppressed." In factorization, the amplitude for $B \to \phi K$ ($K = K_S$ or K^*) has the following structure:

$$A(B \to \phi K) = A_1 + A_2 \ , \tag{4}$$

where

$$\begin{aligned} A_1 &= (B_1 + B_2/N_c) \langle K| \, \bar{s} O_1 b \, |B\rangle \langle \phi| \, \bar{s} O_2 s \, |0\rangle \,, \\ A_2 &= (B_2 + B_1/N_c) \langle K| \, \bar{s} O_{1_F} b \, |B\rangle \langle \phi| \, \bar{s} O_{2_F} s \, |0\rangle \ . \end{aligned} \tag{5}$$

In the above, $\bar{s}O_{1_F} b \bar{s} O_{2_F} s$ is obtained from $\bar{s} O_1 b \bar{s} O_2 s$ by performing a Fierz transformation of the fermions and the colors. The color octet piece is neglected, because it does not lead to the production of a ϕ from the vacuum.

Now, if A_1 or A_2 vanishes, or if $\bar{s}O_{1_F} b \bar{s} O_{2_F} s$ is the same as $\bar{s} O_1 b \bar{s} O_2 s$, then there is only one amplitude, and we can work only with color-allowed operators with a general coefficient – the color-suppressed operators are implicitly included in them. As we show below, this holds for all the operators of Eq. (1). In what follows, the key point is that any S/P operator does not contribute to the decay because it cannot produce a ϕ from the vacuum. However, V/A and T operators do give a nonzero contribution.

- Lorentz structure $(V \pm A) \times (V \pm A)$: $\bar{s}O_{1_F} b \bar{s} O_{2_F} s = \bar{s} O_1 b \bar{s} O_2 s$.
- $(V \pm A) \times (V \mp A)$: Fierz transforms into an S/P operator. Thus, $A_2 = 0$.
- $(S \pm P) \times (S \mp P)$: Fierz transforms into a V/A operator. Thus, $A_1 = 0$.
- $(S \pm P) \times (S \pm P)$: Fierz transforms into a combination of an S/P and a T operator. Thus, $A_1 = 0$ and $A_2 \neq 0$.
- T: Fierz transforms into a combination of an S/P and a T operator. Thus, we effectively have $\bar{s}O_{1_F} b \bar{s} O_{2_F} s = \bar{s} O_1 b \bar{s} O_2 s$.

In all cases, there is only one amplitude in Eq. (5) above, and so the operators of Eq. (1) contain all the "color-suppressed" operators.

We begin by examining the case where a single NP operator is added, contributing to the $\bar{b} \to \bar{s} s \bar{s}$ amplitude. As noted above, this affects both $B \to \phi K^*$ and $B_d^0 \to \phi K_S$, and we compute the order of magnitude of the contribution of each of the NP operators to these decays as follows. Consider $O_{LL}^{V/A}$ (for the orders of magnitude, we ignore the $1/N_c$ coming from the inclusion of the color-suppressed operators):

$$\begin{aligned} \langle \phi K| \, O_{LL}^{V/A} \left| B_d^0 \right\rangle &= \langle \phi K| \, \bar{s} \gamma_\mu (1 - \gamma_5) b \, \bar{s} \gamma^\mu (1 - \gamma_5) s \left| B_d^0 \right\rangle \\ &\equiv \langle \phi K| \, (V - A) \otimes (V - A) \left| B_d^0 \right\rangle \\ &= \langle K| \, V - A \left| B_d^0 \right\rangle_\mu \langle \phi| \, V - A \, |0\rangle^\mu \\ &= \langle K| \, V \left| B_d^0 \right\rangle_\mu \langle \phi| \, V \, |0\rangle^\mu - \langle K| \, A \left| B_d^0 \right\rangle_\mu \langle \phi| \, V \, |0\rangle^\mu \ . \end{aligned} \tag{6}$$

We can now use the factorized matrix elements discussed in appendix A. We obtain

$$
\begin{aligned}
\langle \phi K_S | \, O_{LL}^{V/A} \left| B_d^0 \right\rangle &= 2 f_+ f_\phi m_B p_c = \mathcal{O}(1), \\
\langle \phi K^* | \, O_{LL}^{V/A} \left| B_d^0 \right\rangle \Big|_{\lambda=0} &= -i f_\phi \frac{m_B + m_{K^*}}{2 m_{K^*}} \left\{ (m_B^2 - m_\phi^2 - m_{K^*}^2) A_1 - \frac{4 m_B^2 p_c^2 A_2}{(m_B + m_{K^*})^2} \right\} \\
&= \mathcal{O}(1), \\
\langle \phi K^* | \, O_{LL}^{V/A} \left| B_d^0 \right\rangle \Big|_{\lambda=+} &= -2 i f_\phi V \frac{m_\phi m_B p_c}{m_B + m_{K^*}} + i f_\phi m_\phi (m_B + m_{K^*}) A_1 \\
&= \mathcal{O}((\Lambda_{QCD}/m_b)^2) \\
\langle \phi K^* | \, O_{LL}^{V/A} \left| B_d^0 \right\rangle \Big|_{\lambda=-} &= +2 i f_\phi V \frac{m_\phi m_B p_c}{m_B + m_{K^*}} + i f_\phi m_\phi (m_B + m_{K^*}) A_1 \\
&= \mathcal{O}(\Lambda_{QCD}/m_b) \ , \qquad (7)
\end{aligned}
$$

where p_c is the magnitude of the momentum of final-state particles in the B_d^0 rest frame. The values of f_+, A_1, A_2 and V are given in appendix A. Note: the polarizations in $B \to \phi K^*$ are denoted L (longitudinal) and $\|$, $\perp$ (transverse). However, above we refer to $+$ and $-$ polarizations – the transverse ($A_{\|,\perp}$) and helicity amplitudes ($A_\pm$) are related by $A_{\|,\perp} = (A_+ \pm A_-)/\sqrt{2}$ (and $\bar{A}_{\|,\perp} = \left(\bar{A}_- \pm \bar{A}_+\right)/\sqrt{2}$ for the CP-conjugate amplitudes). The above $B_d^0 \to \phi K$ amplitudes correspond roughly to orders of magnitude $1, 1, \xi^2, \xi$, where $\xi = \mathcal{O}(\Lambda_{QCD}/m_b)$.

	ϕK_S	ϕK^*(L)	$\phi K^*(-)$	$\phi K^*(+)$
$O_{LL}^{V/A}, O_{LR}^{V/A}$	1	1	ξ^2	ξ
$O_{RL}^{V/A}, O_{RR}^{V/A}$	1	1	ξ	ξ^2
$O_{LL}^{S/P}$	ξ^2	ξ	1	ξ^2
$O_{LR}^{S/P}$	1	1	ξ^2	ξ
$O_{RL}^{S/P}$	1	1	ξ	ξ^2
$O_{RR}^{S/P}$	ξ^2	ξ	ξ^2	1
O_L^T	ξ	ξ	1	ξ^2
O_R^T	ξ	ξ	ξ^2	1

Table 2: Relative orders of magnitude of the contribution of NP operators to the amplitudes of $B_d^0 \to \phi K_S$ and the three polarizations of $B \to \phi K^*$ [$\xi = \mathcal{O}(\Lambda_{QCD}/m_b)$].

We have analyzed all NP operators similarly. The results are shown in Table 2. In order to generate large CP asymmetries in $B_d^0 \to \phi K_S$, the contribution of the NP operator must be large [$O(1)$]. This points to the four $O^{V/A}$ operators, $O_{LR}^{S/P}$ or $O_{RL}^{S/P}$. In order to reproduce the f_T/f_L measurement in $B \to \phi K^*$, the NP contribution to a transverse polarization must be large [$O(1)$], while not contributing significantly to the longitudinal polarization. We see that only NP operators of the form $O_{LL}^{S/P}$, $O_{RR}^{S/P}$, O_L^T or O_R^T satisfy this criterion [8].

However, the key point here is that there is no NP operator that significantly affects both $B_d^0 \to \phi K_S$ and a transverse amplitude of $B \to \phi K^*$. We therefore conclude that, as long as $B_d^0 \to \phi K_S$ exhibits large CP-violating effects, there is no single NP operator which can account for the observations in both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ decays. Note: if one assumes that there are no NP signals in this decay, which might be justified with the present errors, then 1-NP-operator solutions are still possible. But if one assumes that there **are** NP signals here, as would clearly be indicated if the errors are reduced by a factor of 2, then 1-NP-operator solutions are not possible.

We now turn to the case where two NP operators are added. Here it is obvious that one of the operators must contribute significantly to $B_d^0 \to \phi K_S$ (6 possibilities), and the other to a transverse amplitude of $B \to \phi K^*$ (4 possibilities). Thus, we must in principle consider 24 pairs of operators. However, this number can be reduced as follows. In any reasonable NP model, if there are only two new operators, these are typically both of the V/A, S/P or T variety. There are no pairs of V/A or T operators which give large contributions to both decays, so there are only 4 pairs of NP operators to examine: ($O_{LL}^{S/P}$, $O_{LR}^{S/P}$), ($O_{LL}^{S/P}$, $O_{RL}^{S/P}$), ($O_{RR}^{S/P}$, $O_{LR}^{S/P}$) and ($O_{RR}^{S/P}$, $O_{RL}^{S/P}$). The results of Table 2 give only the general size of contributions, so it is necessary to perform a fit to see which pairs of NP operators can account for the observations in both decays, and to what extent. We do this below.

BR	$9.8^{+0.7}_{-0.6} \times 10^{-6}$
A_{CP}	0.01 ± 0.05
f_L	0.480 ± 0.030
$f_\perp$	0.241 ± 0.029

Table 3: Measurements of $B \to \phi K^{*0}$. Included are the branching fraction (BR), the direct CP symmetry (A_{CP}), and the fraction of longitudinal and $\perp$ decays, f_L and $f_\perp$ [9, 11, 16].

The fit includes the three observables of $B_d^0 \to \phi K_S$ shown in Table 1: BR, S_{CP}, A_{CP}. It also includes four observables of $B \to \phi K^*$: BR, A_{CP}, f_L, $f_\perp$. The latest values are given in Table 3[5]. (There are other measurements of $B \to \phi K^*$, but they are not used in this paper.)

The NP contributions to the above quantities are taken into account as follows. We are considering the effect of the SM and two NP operators (we generally refer to them as O_1 and O_2). The strength of the NP is parametrized by unknown complex coefficients (referred to as C_1 and C_2). The SM piece is calculable within QCD factorization (QCDf) [17], and we take the value of its contribution from there. C_1 and C_2 generally each have a magnitude, a weak phase, and a strong phase. However, in Ref. [18], it was shown that the NP strong phases are negligible compared to that of the (dominant)

[5]In Table 3 it is given that $f_L = 0.48$, $f_\perp = 0.24$. The f_i are defined such that $f_\parallel = 1 - f_L - f_\perp = 0.28$, so that indeed $f_T/f_L = (f_\perp + f_\parallel)/f_L \simeq 1$.

SM contribution. Thus, we have only four free parameters: two NP magnitudes and two NP weak phases of C_1 and C_2. The decay amplitudes A^λ for a given helicity λ can be written in terms of these free parameters simply by computing the matrix elements for $B_d^0 \to \phi K_S$ and each polarization of $B \to \phi K^*$. In general, they take the form

$$\begin{aligned} A^\lambda &= A^\lambda_{SM} + \langle \phi K | \, C_1 O_1 + C_2 O_2 \left| B_d^0 \right\rangle \Big|_\lambda \\ &\equiv A^\lambda_{SM} + C_1 A_1^\lambda + C_2 A_2^\lambda \ , \end{aligned} \tag{8}$$

where $A_{1,2}$ are the factorized matrix elements given in appendix A. The CP-conjugate amplitudes $\bar{A}$ are obtained by changing the sign of the unknown weak phases in C_1 and C_2.

All observables can be expressed in terms of the amplitudes A^λ. The branching ratio is given by

$$\mathrm{BR} = \frac{\tau_B p_c}{8\pi\hbar m_B^2} \sum_\lambda |A^\lambda|^2 \ , \tag{9}$$

where τ_B is the lifetime of the B_d^0 meson. This applies to both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$. The time-independent CP asymmetry is given by

$$A_{CP} = -\frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \ . \tag{10}$$

For $B_d^0 \to \phi K_S$, A is the amplitude and $\bar{A}$ is its CP-conjugate. For $B \to \phi K^*$, $|A|^2 = \sum_\lambda |A^\lambda|^2$. The time-dependent CP-asymmetry in $B_d^0 \to \phi K_S$ is given by

$$S_{CP} = -2\,\frac{\mathrm{Im}(e^{-2i\beta} A^* \bar{A})}{|A|^2 + |\bar{A}|^2} \ . \tag{11}$$

Finally, the helicity fractions of $B \to \phi K^*$ are defined as usual by

$$f^B_{L,||,\perp} = \frac{|A^{L,||,\perp}|^2}{|A^L|^2 + |A^{||}|^2 + |A^\perp|^2} \ , \tag{12}$$

and similarly for the CP-conjugates $f^{\bar{B}}_{L,||,\perp}$. Combining them, $f_{L,||,\perp}$ are defined by

$$f_{L,||,\perp} = \frac{1}{2}(f^B_{L,||,\perp} + f^{\bar{B}}_{L,||,\perp}) \ . \tag{13}$$

The 7 observables can therefore be expressed in terms of the 4 free parameters of C_1 and C_2. Thus, it is possible to perform a fit to obtain the preferred values of these parameters, and to determine whether it is possible to account for the data with the addition of certain NP operators. However, there is a complication in all of this. In the SM, the decays $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ are dominated by the $\bar{b} \to \bar{s}$ penguin amplitude, P'. P' is actually composed of three pieces, P'_u, P'_c and P'_t, where

the subscript refers to the internal quark in the loop (the pieces $P'_{u,c}$ are rescattering amplitudes generated mainly from tree-level operators). We can write

$$\begin{aligned} P' &= V_{ub}^* V_{us}\, P'_u + V_{cb}^* V_{cs}\, P'_c + V_{tb}^* V_{ts}\, P'_t \\ &\simeq V_{cb}^* V_{cs}\, (P'_c - P'_t) \; . \end{aligned} \qquad (14)$$

In writing the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa matrix to eliminate the $V_{tb}^* V_{ts}$ term, and we have dropped the $V_{ub}^* V_{us}$ term since $|V_{ub}^* V_{us}| \ll |V_{cb}^* V_{cs}|$. As the weak phase of $V_{cb}^* V_{cs}$ is zero, P' has only its magnitude and a strong phase. It is these quantities that are calculated in QCDf. Unfortunately, as we see below, the QCDf results are not very precise.

The QCDf calculation is treated by applying Refs. [17, 19, 20] straightforwardly. For $B_d^0 \to \phi K_S$,

$$\frac{P'_c - P'_t}{A_{\phi K_S}} = \alpha_3^c + \alpha_4^c - \frac{1}{2}\alpha_{3,EW}^c - \frac{1}{2}\alpha_{4,EW}^c \equiv \mathcal{P}_{\phi K_S} \; , \qquad (15)$$

where the $A_{\phi K_S}$ are form factors as defined in appendix A, and the α's are defined in Ref. [19]. (Note: the above formula could contain β terms. However, we have neglected all such pieces, consistent with our assumption that PA is not present.) For $B \to \phi K^*$, the SM penguin has the same form, but with explicit polarization dependence λ:

$$\frac{P_c^{\prime\lambda} - P_t^{\prime\lambda}}{A_{\phi K^*}^{\lambda}} = \alpha_3^{c,\lambda} + \alpha_4^{c,\lambda} - \frac{1}{2}\alpha_{3,EW}^{c,\lambda} - \frac{1}{2}\alpha_{4,EW}^{c,\lambda} \equiv \mathcal{P}_{\phi K^*}^{\lambda} \; . \qquad (16)$$

The α^{λ}'s are defined in Ref. [20]. Several inputs are required in order to get magnitudes and strong phases. For quark masses, BBNS values [17] were used, allowing them to vary within a range of 1σ. For meson masses, Particle Data Group values [9] were used, fixed at their central value. Wilson coefficients were calculated using Refs. [21, 22], with the renormalization scale μ allowed to vary within $[m_b/2, m_b]$. For decay constants, values from Table 10 in appendix A were used, within a range of 1σ. For form factors, fixed values of Table 9 in appendix A were used, but we studied each of the three cases (minimal, central and maximal values). The allowed ranges of the SM penguin amplitudes are summarized in Table 4. The SM penguin of negative helicity is neglected because of the small form factors ($\mathcal{P}_{\phi K^*}^{-} = \bar{\mathcal{P}}_{\phi K^*}^{+} \simeq 0$). As expected, form factors have little impact on the values of the SM penguin amplitudes $\mathcal{P}_{\phi K_S}$ and $\mathcal{P}_{\phi K^*}^{\lambda}$, since they contribute at subleading order. Numerical variations are mainly due to the random scanning of the parameter space.

Ideally, in order to take into account the ranges of the QCDf determinations, one would scan over the allowed regions of all magnitudes and strong phases. For each set of SM values, the χ^2 would be evaluated. In this way, we could find the best fit (i.e. smallest value of χ^2_{min}) for each of the 2-NP-operator solutions. Unfortunately, this is not possible, as the space of SM values is too large (e.g. if we take 10 SM values/region,

	Minimal	Central	Maximal
$\lvert\mathcal{P}_{\phi K_S}\rvert$	$[0.031, 0.064]$	$[0.031, 0.062]$	$[0.031, 0.060]$
$\mathrm{Arg}(\mathcal{P}_{\phi K_S})$(rad)	$[3.2, 3.6]$	$[3.2, 3.6]$	$[3.2, 3.6]$
$\lvert\mathcal{P}^0_{\phi K^*}\rvert$	$[0.025, 0.036]$	$[0.026, 0.036]$	$[0.027, 0.037]$
$\mathrm{Arg}(\mathcal{P}^0_{\phi K^*})$(rad)	$[3.4, 3.6]$	$[3.4, 3.6]$	$[3.4, 3.6]$
$\lvert\mathcal{P}^+_{\phi K^*}\rvert$	$[0.031, 0.062]$	$[0.033, 0.062]$	$[0.033, 0.061]$
$\mathrm{Arg}(\mathcal{P}^+_{\phi K^*})$(rad)	$[3.0, 3.4]$	$[3.0, 3.4]$	$[3.0, 3.4]$

Table 4: Allowed ranges for SM penguin magnitudes and strong phases according to QCDf, for the three sets of form-factor values.

we would require 10^6 χ^2 evaluations). As a compromise, we have adopted the following procedure: we fix the SM strong phases to their central values, and scan over the SM magnitudes. However, we have checked what happens when we take different values for the strong phases. We find that the χ^2 numbers can change quite a bit, but a bad fit cannot be turned into a good fit.

Operators	Minimal	Central	Maximal
$(O^{S/P}_{LL}, O^{S/P}_{RL})$	2.6 (45.7%)	2.8 (42.4%)	3.1 (37.6%)
$(O^{S/P}_{LL}, O^{S/P}_{LR})$	1.4 (70.6%)	1.3 (72.9%)	1.3 (72.9%)
$(O^{S/P}_{RR}, O^{S/P}_{RL})$	1.9 (59.3%)	1.7 (63.7%)	1.6 (65.9%)
$(O^{S/P}_{RR}, O^{S/P}_{LR})$	1.7 (63.7%)	1.7 (63.7%)	1.6 (65.9%)
$(O^{V/A}_{LL(RR)}, O^{V/A}_{RL(LR)})$	15.7 (0.13%)	10.6 (1.4%)	7.1 (6.9%)
(O^{T}_{R}, O^{T}_{L})	3.6 (30.8%)	3.6 (30.8%)	3.9 (27.2%)

Table 5: Best-fit results (χ^2_{min}) for pairs of NP operators, with present-day errors on S_{CP} and A_{CP} in $B^0_d \to \phi K_S$. The calculation was done for the three sets of form factors (minimal, central and maximal).

The results are shown in Tables 5 (current errors on S_{CP} and A_{CP}) and 6 (errors on S_{CP} and A_{CP} reduced by a factor of 2). Here we present the smallest value of χ^2_{min} (best fit) for each of the 2-NP-operator solutions. (The number in parentheses indicates the quality of the fit, and depends on χ^2_{min} and *d.o.f.* individually. 50% or more is a good fit; fits which are substantially less than 50% are poorer.) In all cases, the worst fit is given by a large value of χ^2_{min}, with a 0% quality of fit.

From Table 5, we see that, with current errors on S_{CP} and A_{CP}, all four S/P 2-NP-operator solutions give good fits to the $B^0_d \to \phi K_S$ and $B \to \phi K^*$ data. We also show the fit results for V/A and T 2-NP-operator solutions. We see that the V/A solution gives a very poor fit, but the T solution, while not as good as any of the S/P pairs, is still acceptable.

If the errors on S_{CP} and A_{CP} are reduced by a factor of 2 (Table 6), we find that no S/P 2-NP-operator hypothesis is an excellent fit to the data. On the other hand,

Operators	Minimal	Central	Maximal
$(O_{LL}^{S/P}, O_{RL}^{S/P})$	6.3 (9.8%)	7.4 (6.0%)	8.6 (3.5%)
$(O_{LL}^{S/P}, O_{LR}^{S/P})$	4.3 (23.1%)	4.0 (26.1%)	3.9 (27.2%)
$(O_{RR}^{S/P}, O_{RL}^{S/P})$	5.2 (15.8%)	5.8 (12.2%)	5.6 (13.3%)
$(O_{RR}^{S/P}, O_{LR}^{S/P})$	4.9 (17.9%)	4.7 (19.5%)	4.5 (21.2%)
$(O_{LL(RR)}^{V/A}, O_{RL(LR)}^{V/A})$	20.3 (0.01%)	15.9 (0.12%)	10.9 (1.2%)
(O_R^T, O_L^T)	13.7 (0.33%)	13.5 (0.37%)	14.0 (0.29%)

Table 6: Best-fit results (χ^2_{min}) for pairs of NP operators, in the scenario in which the errors on S_{CP} and A_{CP} in $B_d^0 \to \phi K_S$ are reduced by a factor of 2. The calculation was done for the three sets of form factors (minimal, central and maximal).

none of them is ruled out, either. The most that one can say is that $(O_{LL}^{S/P}, O_{RL}^{S/P})$ and $(O_{RR}^{S/P}, O_{RL}^{S/P})$ are disfavored, but even this is not very strong. On the other hand, in this case both V/A and T 2-NP-operator solutions are essentially ruled out.

In both error scenarios, it is the large direct CP-asymmetry measurement in $B_d^0 \to \phi K_S$ which is hardest to accommodate. It will be important to pay attention to this observable in the future to determine which NP solutions are viable.

The fact that the best fit and worst fit have substantially different χ^2_{min} shows that the contribution from the SM is significant, and that all χ^2 ranges would be reduced quite a bit with an improved determination of the SM values. In fact, one could easily obtain poor fits for all pairs of NP operators (as well as the SM).

Above, we have shown that all 2-NP-operator solutions involving S/P operators are viable, but those which contain only V/A or T operators are disfavored or ruled out. Obviously, any realistic NP model which contains more than 2 operators – and most do – will also be allowed, as long as the observations in both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ decays are explained. In Table 7, we show which types of operators are present for some simple NP models. Even though models with supersymmetry[6] or extra Z' bosons typically generate several operators, they are all of V/A type. As such, they cannot explain both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ data. On the other hand, the two-Higgs-doublet model is favored because it contains only S/P operators (perhaps all 4 pairs), and can potentially accomodate both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$.

Models	V/A	S/P	T
Supersymmetry [23, 24, 25]	$\times$		
Two Higgs doublets [26, 27]		$\times$	
Extra Z' bosons [28]	$\times$		

Table 7: Summary of operator content for some simple NP models.

Even with the assumption of reduced errors on the CP-violating observables in

[6]Models with supersymmetry generate the $\bar{b} \to \bar{s} s \bar{s}$ transition mainly through squark-gluino loops.

$B_d^0 \to \phi K_S$, all four S/P 2-NP-operator solutions are allowed. This raises the obvious question: is there any way to distinguish these solutions? Clearly, smaller errors on the experimental measurements and/or the theoretical determination of the SM contribution can help. With these, it may be that the χ^2_{min} of one solution is strongly preferred over that of the other three.

However, there is an additional possibility. In any $B \to V_1 V_2$ decay, one can construct the *triple product* (TP). In the rest frame of the B, the TP takes the form $\vec{q} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$, where $\vec{q}$ is the momentum of one of the final vector mesons, and $\vec{\varepsilon}_1$ and $\vec{\varepsilon}_2$ are the polarizations of V_1 and V_2. There are two TP's, which can be written [29]

$$A_T^{(1)} \equiv \frac{\mathrm{Im}(A_\perp A_0^*)}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2} , \qquad A_T^{(2)} \equiv \frac{\mathrm{Im}(A_\perp A_{||}^*)}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2} . \tag{17}$$

The $\bar{A}_T^{(1,2)}$ for $\bar{B}$ decays are defined similarly. The TP asymmetry is defined by[7]

$$\mathcal{A}_T^{(i)} = \frac{A_T^{(i)} - \bar{A}_T^{(i)}}{2} . \tag{18}$$

The "fake" TP asymmetry $\tilde{\mathcal{A}}_T^{(i)}$ is given by the same definition, but the TP's are added rather than subtracted. Note that the fake TP asymmetry can be nonzero even if CP is conserved.

Operators	$\mathcal{A}_T^{(1)}$	$\tilde{\mathcal{A}}_T^{(1)}$	$\mathcal{A}_T^{(2)}$	$\tilde{\mathcal{A}}_T^{(2)}$
$(O_{LL}^{S/P}, O_{RL}^{S/P})$	$[-0.30,\ -0.27]$	$[0.030,\ 0.062]$	$[0.16,\ 0.22]$	$[-0.006,\ -0.004]$
$(O_{LL}^{S/P}, O_{LR}^{S/P})$	$[0.29,\ 0.32]$	$[-0.008,\ 0.014]$	$[-0.17,\ -0.14]$	$[-0.003,\ 0.000]$
$(O_{RR}^{S/P}, O_{RL}^{S/P})$	$[0.26,\ 0.28]$	$[-0.099,\ 0.056]$	$[-0.037,\ 0.090]$	$[-0.004,\ 0.001]$
$(O_{RR}^{S/P}, O_{LR}^{S/P})$	$[-0.33,\ -0.31]$	$[-0.036,\ -0.011]$	$[-0.001,\ 0.000]$	0.000

Table 8: Predictions of all S/P 2-NP-operator solutions of Table 5 for the central values of the real and fake TP asymmetries in $B \to \phi K^*$. The ranges of TP-asymmetry predictions correspond to the full variation of form-factor values.

The above applies to $B \to \phi K^*$. In Table 8 we present the central values of $\mathcal{A}_T^{(1,2)}$ and $\tilde{\mathcal{A}}_T^{(1,2)}$, calculated for each of the S/P solutions shown in Table 5. The ranges correspond to the sets of form-factor values varying from minimal to maximal. We do not include errors because they are very large with the current data. In any case, our point in presenting the results of Table 8 is the following. It is clear that different S/P 2-NP-operator solutions lead to very different patterns of central values of predictions for the TP asymmetries. This emphasizes the usefulness of TP's for distinguishing

[7] Note: in contrast to Ref. [29], this definition involves a subtraction rather than an addition. This is because we have defined $A_{\perp,||}$ and $\bar{A}_{\perp,||}$ in such a way that $\mathcal{A}_T^{(i)}$ is zero in the absence of CP violation.

the various NP solutions, and we strongly encourage experimentalists to make such measurements.

In summary, a "polarization puzzle" has been observed in $B \to \phi K^*$, namely that the fraction of transversely-polarized decays is about equal to that of longitudinally-polarized decays, in contrast to the expectations of the naive standard model (SM). In this paper, we explore the new-physics (NP) solution to this puzzle. We first note that any NP explanation must also be consistent with the observations in $B_d^0 \to \phi K_S$. This decay is particularly intriguing since the present measurements of CP-violating asymmetries are in disagreement with the SM. On the other hand, the errors are still sufficiently large that this discrepancy is not statistically significant. As such, any constraints on NP in $B \to \phi K^*$ are not that stringent. For this reason, we also perform the analysis with the assumption that future measurements will show a greater statistical discrepancy. That is, we use the central values of the CP-asymmetry measurements, but take the errors to be reduced by a factor of 2.

We consider 10 NP operators, of types S/P, V/A and T. We first show that, as long as $B_d^0 \to \phi K_S$ exhibits large CP-violating effects, no single NP operator can explain the data in both $B \to \phi K^*$ and $B_d^0 \to \phi K_S$ decays. Turning to 2-NP-operator solutions, it is clear that one of the operators must contribute significantly to $B_d^0 \to \phi K_S$, and the other to a transverse amplitude of $B \to \phi K^*$. In any realistic NP model the two operators are typically both of the V/A, S/P or T type. However, no pairs of V/A or T operators give large contributions to both decays, so that only 4 pairs of S/P operators need be considered. We have performed fits to several observables in $B \to \phi K^*$ and $B_d^0 \to \phi K_S$ decays, and find that all four 2-NP-operator solutions are allowed. (We also show explicitly that the V/A and T solutions are disfavored or ruled out.)

One can distinguish among the solutions in several ways. If the experimental errors on future measurements are improved, one solution might be preferred. Alternatively, the theoretical uncertainty can be reduced if the SM contribution to $B \to \phi K^*$ and $B_d^0 \to \phi K_S$ is better determined. Finally, the four solutions predict a very different pattern of triple-product asymmetries in $B \to \phi K^*$. Their measurement could help distinguish among the possible NP solutions.

Finally, any realistic NP model which contains more than two operators will also be allowed, as long as the measurements in both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ decays are explained. However, models with supersymmetry or extra Z' bosons contain only operators of V/A type, and therefore cannot explain both $B_d^0 \to \phi K_S$ and $B \to \phi K^*$ data. In contrast, the two-Higgs-doublet model has only S/P operators, and is thus favored.

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A Calculation of factorized matrix elements

For NP, we work within the framework of naive factorization. Since LO contributions of NP are expected to be subleading compared with the SM, NLO contributions of NP can be safely neglected. Then, for a general effective four-quark operator $O \sim X \otimes Y$ ($X, Y = S, P, V, A, T$ or $T\gamma_5$), the matrix element is assumed to factorize as

$$\langle \phi K | \, O \, | B \rangle \to \langle K | \, X \, | B \rangle \, \langle \phi | \, Y \, | 0 \rangle \ , \tag{19}$$

where K stands for K_S or K^*. $\langle K | \, X \, | B \rangle$ is calculable using known form factors; $\langle \phi | \, Y \, | 0 \rangle$ is calculable using the ϕ-meson decay constants.

For $B \to K$ form factors, we use definitions from Refs. [30, 31]:

$$\begin{aligned}
\left\langle K(p) \right| V \left| \bar{B}(p_B) \right\rangle_\mu &= f_+(s) \left\{ (p_B + p)_\mu - \frac{m_B^2 - m_K^2}{s} q_\mu \right\} \\
&\quad + \frac{m_B^2 - m_K^2}{s} f_0(s) q_\mu \ , \\
\left\langle K(p) \right| T(1 \pm \gamma_5) \left| \bar{B}(p_B) \right\rangle_{\mu\nu} q^\nu &= \left\langle K(p) \right| T \left| \bar{B}(p_B) \right\rangle_{\mu\nu} q^\nu \\
&= i \left\{ (p_B + p)_\mu s - q_\mu (m_B^2 - m_K^2) \right\} \frac{f_T(s)}{m_B + m_K} \ , \\
\left\langle K^*(p, \epsilon) \right| V \pm A \left| \bar{B}(p_B) \right\rangle_\mu &= \pm i \epsilon_\mu^* (m_B + m_{K^*}) A_1(s) \\
&\quad \mp i (p_B + p)_\mu (\epsilon^* \cdot p_B) \frac{A_2(s)}{m_B + m_{K^*}} \\
\mp i q_\mu (\epsilon^* \cdot p_B) \frac{2 m_{K^*}}{s} (A_3(s) - A_0(s)) &+ \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_B^\rho p^\sigma \frac{2 V(s)}{m_B + m_{K^*}} \\
\left\langle K^*(p, \epsilon) \right| T(1 \pm \gamma_5) \left| \bar{B}(p_B) \right\rangle_{\mu\nu} q^\nu &= i \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_B^\rho p^\sigma 2 T_1(s) \\
&\quad \pm T_2(s) \left\{ \epsilon_\mu^* (m_B^2 - m_{K^*}^2) - (\epsilon^* \cdot p_B)(p_B + p)_\mu \right\} \\
&\quad \pm T_3(s) (\epsilon^* \cdot p_B) \left\{ q_\mu - \frac{s}{m_B^2 - m_{K^*}^2} (p_B + p)_\mu \right\} \ ,
\end{aligned} \tag{20}$$

with $q = p_B - p$ and $s = q^2$. Values of the form factors are tabulated in Tables 3, 4 and 5 of Ref. [30] with minimal and maximal values and s dependance. Using these, we have calculated them for the case of $s = m_\phi^2$ (see Table 9).

The ϕ vector-meson decay constants are defined by [30, 31]

$$\begin{aligned}
\langle \phi(q, \epsilon) | \, V \, | 0 \rangle^\mu &= f_\phi m_\phi \epsilon^{*\mu} \ , \\
\langle \phi(q, \epsilon) | \, T \, | 0 \rangle^{\mu\nu} &= -i f_\phi^\perp (\epsilon^{*\mu} q^\nu - \epsilon^{*\nu} q^\mu) \ ,
\end{aligned} \tag{21}$$

which imply

$$\langle \phi(q, \epsilon) | \, T\gamma_5 \, | 0 \rangle^{\mu\nu} = -\frac{1}{2} f_\phi^\perp \epsilon^{\mu\nu\rho\sigma} (\epsilon_\rho^* q_\sigma - \epsilon_\sigma^* q_\rho) \ . \tag{22}$$

	Minimal value	Central value	Maximal value
f_+	0.295	0.337	0.391
f_0	0.286	0.327	0.379
f_T	0.319	0.375	0.446
A_1	0.301	0.345	0.393
A_2	0.258	0.295	0.333
A_0	0.437	0.498	0.750
V	0.423	0.483	0.579
T_1	0.355	0.402	0.463
T_2	0.341	0.387	0.445
T_3	0.245	0.272	0.307

Table 9: Values of $B \to K$, $B \to K^*$ form factors for $s = m_\phi^2$ following Ref. [30] (calculated in the QCD light-cone sum-rules approach at the scale $\mu = m_b$).

Values of the decay constants are tabulated in Table 10.

M	ϕ [MeV]	B [MeV]	K [MeV]	K^* [MeV]
f_M	215 ± 5	200 ± 25	160	220 ± 5
$f_M^\perp$	186 ± 9	–	–	185 ± 10

Table 10: Values of decay constants for mesons.[31]

In order to calculate factorized matrix elements, we define the four-momemta

$$p_B = (m_B, 0, 0, 0)\ , \qquad p_{K,K^*} = (E_{K,K^*}, 0, 0, -p_c)\ , \qquad p_\phi = (E_\phi, 0, 0, p_c)\ , \tag{23}$$

and polarization 4-vectors

$$\begin{aligned} \epsilon^0_{K^*} &= \frac{1}{m_{K^*}}(p_c, 0, 0, -E_{K^*})\ , & \epsilon^\pm_{K^*} &= \frac{1}{\sqrt{2}}(0, \mp 1, +i, 0)\ , \\ \epsilon^0_\phi &= \frac{1}{m_\phi}(p_c, 0, 0, E_\phi)\ , & \epsilon^\pm_\phi &= \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)\ , \end{aligned} \tag{24}$$

in the B-meson rest frame. From here it is straightforward to calculate factorized matrix elements using the above. Those which are non-zero are

$$\begin{aligned} \langle K | V \left| \bar{B} \right\rangle_\mu \langle \phi | V | 0 \rangle^\mu &= 2 f_+ f_\phi m_B p_c\ , \\ \langle K | T \left| \bar{B} \right\rangle_{\mu\nu} \langle \phi | T | 0 \rangle^{\mu\nu} &= 4 f_T f_\phi^\perp \frac{m_\phi m_B p_c}{m_B + m_K}\ , \end{aligned} \tag{25}$$

for $B_d^0 \to \phi K_S$ and

$$\langle K^* | T \left| \bar{B} \right\rangle_{\mu\nu} \langle \phi | T | 0 \rangle^{\mu\nu} \Big|_{\lambda=\pm} = \langle K^* | T\gamma_5 \left| \bar{B} \right\rangle_{\mu\nu} \langle \phi | T\gamma_5 | 0 \rangle^{\mu\nu} \Big|_{\lambda=\pm}$$

$$
\begin{aligned}
&= \mp 4 i f_\phi^\perp T_1 m_B p_c \ , \\
\left\langle K^* \right| T \left| \bar{B} \right\rangle_{\mu\nu} \left\langle \phi \right| T\gamma_5 \left| 0 \right\rangle^{\mu\nu} \Big|_{\lambda=0} &= i \frac{f_\phi^\perp}{m_\phi m_{K^*}} \Big\{ -T_2 (m_B^2 - m_\phi^2 - m_{K^*}^2)(m_B^2 - m_{K^*}^2) \\
&\quad +4 p_c^2 m_B^2 \left[T_2 + T_3 m_\phi^2 / (m_B^2 - m_{K^*}^2) \right] \Big\} \ , \\
\left\langle K^* \right| T \left| \bar{B} \right\rangle_{\mu\nu} \left\langle \phi \right| T\gamma_5 \left| 0 \right\rangle^{\mu\nu} \Big|_{\lambda=\pm} &= 2 i f_\phi^\perp T_2 (m_B^2 - m_{K^*}^2) \ , \\
\left\langle K^* \right| V \left| \bar{B} \right\rangle_{\mu} \left\langle \phi \right| V \left| 0 \right\rangle^{\mu} \Big|_{\lambda=\pm} &= \mp 2 i f_\phi V \frac{m_\phi m_B p_c}{m_B + m_{K^*}} \ , \\
\left\langle K^* \right| A \left| \bar{B} \right\rangle_{\mu} \left\langle \phi \right| V \left| 0 \right\rangle^{\mu} \Big|_{\lambda=0} &= i f_\phi \frac{m_B + m_{K^*}}{2 m_{K^*}} \Big\{ (m_B^2 - m_\phi^2 - m_{K^*}^2) A_1 \\
&\quad - \frac{4 m_B^2 p_c^2 A_2}{(m_B + m_{K^*})^2} \Big\} \ , \\
\left\langle K^* \right| A \left| \bar{B} \right\rangle_{\mu} \left\langle \phi \right| V \left| 0 \right\rangle^{\mu} \Big|_{\lambda=\pm} &= -i f_\phi m_\phi (m_B + m_{K^*}) A_1 \qquad (26)
\end{aligned}
$$

for $B \to \phi K^*$. All of this allows us to calculate the entries in Table 2.